A Review of Multi Objective Optimization

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***Abstract*— Merging systems, enhancing inter-disciplinary relations and increasing needs require multi objectives rather than a single objective in the optimization problems nowadays. However, the objectives are frequently conflicting. When an objective is improved, the other objective(s) may deteriorate. In the multi-objective optimization problems (MOOPs), the aim is to come up with the best solutions that can be an alternative for each other in terms of objective function values under the constraints caused by various reasons. During the last two decades, MOOPs and solution methods have been studied with great interest. It is possible to come across a MOOP in almost every discipline in the literature. MOOPs have been modelled and solved not only in the fields with more applications such as production, management, business administration, marketing, transportation and finance but also in the basic sciences such as chemistry, maths and statistics. Solution of MOOPs requires the simultaneous optimization of conflicting multi objectives. In MOOPs, an optimal solution set on which a compromise is reached among the conflicting objectives is obtained. In this study, the articles on multi-objective optimization written in 2015 and later are analysed and 61 articles are chosen among them. Classical and heuristic methods implemented for the solution of MOOPs presented in these articles are mentioned. The articles are classified according to their subject areas. The methodology used in each article is identified. According to their implementation areas, the multi-objective optimization methods and the areas they are implemented the most are discussed. The areas to be focused on in the future studies to obtain more robust results in the optimization are identified.**

***Keywords*— multi-objective optimization, Pareto optimal set, genetic algorithm methods, particle swarm optimization**

I. INTRODUCTION

Optimization can be described as obtaining the best solution in line with the constraints identified for objective(s) desired to be reached. First, decision variable is identified and then, objective function of the problem is formed in the solution of an optimization problem. Constraint function of the problem, if available, is described and the appropriate solution method to be chosen is applied according to optimization classification. Although the majority of studies on optimization focus on a single objective, optimization problems we come across in practice are based on multi-objective optimization.

In this study, articles included in the literature in the last 15 years and referred more than other articles written in the same year are analysed in terms of subjects, methodologies and results for Multi-objective optimization. These concepts in the literature are handled under separate headings. Multi-objective optimization methods can be divided into two groups as classical and heuristic methods. In this study, we explain the methods mostly used or referred in the articles. Each article is analysed in terms of its methodology under the relevant heading. Other unexplained methods and new approaches related to multi-objective optimization are analysed under the same heading. Methods chosen in the articles are analysed along with the reasons in the conclusion part. Success criteria of the studies conducted in special areas are mentioned. Subjects are suggested for future studies to obtain more solid results from multi-objective optimization.

II. MULTI-OBJECTIVE OPTIMIZATION

The main difference between single objective problems and multi-objective problems is that; there is a single optimal solution or alternative optimal solutions in single objective problems while there is not any solution simultaneously delivering the best value for each objective in multi-objective problems. Since all objectives are important in these problems, looking for a solution only for one objective is wrong. Improvements in an objective cause deterioration in at least one of the other objectives due to conflicting objectives. Trade-off amounts are obtained among identified solutions depending on their function values because of conflicting objectives. Acceptable trade-off amounts depend on decision makers. In this case, there is generally not a certain optimal solution in multi-objective problems. Some of the solutions obtained and traded-off are presented to the decision maker as the best solutions.

A general multi-objective optimization problem includes n parameters (decision variable), k objective functions and a group of m constraints. Objective functions and constraints are written as the function of decision variables. Optimization process can be formulated as

$$\min \quad y = f(x) = \left(f_1(x), f_2(x), \dots, f_k(x)\right)$$
$$e(x) = \left(e_1(x), e_2(x), \dots, e_m(x)\right) \leq 0$$

$$x = (x_1, x_2, \ldots, x_n) \in X \quad (1)$$
$$y = (y_1, y_2, \ldots, y_k) \in Y$$

. Here it is appropriate to describe y as the decision vector, y as the objective vector, X as the decision space and Y as the objective space. Constraints e(x)≤0 show accessible solution set for the described problem. [1].

A. Dominance

Let a, b ∈ X be two separate solution vectors of a multi-objective optimization. In a problem with maximization type objective functions, solution a dominates solution b if $f_j(a) > f_j(b)$ j ∈ $(1,2,\ldots n)$ is the case for each i $f_i(a) \geq f_i(b)$ $i \in (1,2,\ldots n)$ and at least one j. In the cases without dominance, a and b solutions cannot dominate each other for at least one i $f_i(a) < f_i(b)$ and at least one j $f_j(a) > f_j(b)$.[2]

B. Pareto Optimal Concept

A solution can be the best, the worst or equal to other solutions depending on its objective values. The best solution means a solution which is not the worst solution in any objective and is better than the other solutions in at least one objective. Pareto-Optimal solution is the solution not dominated by another solution in the search space [2].

Considering all objective functions, the set of all dominant solutions is called Pareto Front. The real aim of multi-objective optimization is to find or approximate to the Pareto front and enable a fair distribution over this front.

III. Classical Methods for Multi-Objective Problems

These methods are effective if the structure of multi-objective decision making problem aimed to be solved is appropriate and finding the best solution is possible if available. Regarding the studies using classical optimization techniques in general, Diakaki et al [38] evaluated the success of classical optimization methods in energy efficiency of buildings. They analysed the suitability of multi-objective optimization techniques in the solution of problems related to energy efficiency of buildings. They observed it in the practices of a simple example. Ren et al [48] designed a modelling for the distributed energy resource (DER) using multi-objective linear programming (MOLP). This modelling was implemented within the scope of a case study in a eco-campus in Japan. Giarola et al [53] realized bio-refineries spatial design with Mixed Integer Linear Programming framework. In this way, they succeeded in simultaneously optimizing economic and environmental performances. Asadi et al [56] presented a multi-objective optimization model with Thebycheff formulation in order to reduce energy cost of buildings. They applied the model on a house by conducting required renovations and showed the feasibility of these approaches. Karande and Chakraborty [60] introduced a new approach for material selection problems in product designs. They developed a multi-objective optimization on the basis of ratio analysis (MOORA) independent from weights of selection criteria and normalization procedures of decision matrices. Najafi et al [63] suggested a source optimization model using classical optimization methods in order to provide the best immediate aid possible in post-earthquake response. Ehrgott et al [79] presented Minmax Robustness approach to obtain more robust results in classical optimization methods. They tried their approach on many linear and quadratic programming. Nariman-Zadeh et al [45] suggested a new multi-objective uniform-diversity genetic algorithm (MUGA) procedure based on developed ε-elimination for vehicle vibration systems.

A. Weighted-Sum

In this method, objectives are given weights, these weights are added together and a single weighted objective function is obtained. Weights are determined according to the importance attached to the objective by the decision maker. Optimal result can be obtained using the function composed of weighted objectives. Determining the weights can be difficult due to the lack of information on the problem during implementation of the method. If the problem is convex and the weights are positive, Pareto optimal solution is obtained [4]. In the other cases, in other words; if the problem is concave and the weights are not positive, it is wrong to look for a solution using this method.

Considering the studies using weight-sum; Altiparmak et al [29] developed the weight-sum approach in the design of supply chains network and a procedure finding Pareto optimal solutions. An experimental study using actual data from a company, which is a producer of plastic products in Turkey, was carried out. Xiang et al [44] came up with the multi-objective optimization (MOO) formulation for human lifting simulation with weighted-sum approach. They observed that the results are more sensitive compared to single objective optimization. Marler and Arora [46] investigated the impacts of weight selection in the solution stages of weighted-sum method and tried to solve the problem without knowing the weights in advance. Devarajan et al [61] presented an optimization modelling using weighted-sum in order to reduce the cost of wireless communication. They observed the results of the approach on simulation. Gjorgiev and Cepin [62] suggested an optimization approach using weighted-sum method for the combined economic-environmental power dispatch problem. They tried the algorithm in different power systems and they compared the results with multi-objective differential evolution (MODE) and NSGA- II. Sun et al [83] presented optimization modellings for patient and resource allocation among hospitals during a pandemic influenza outbreak. Weighted-sum was used in mathematical modellings.

IV. Multi-Objective Decision Making Problems Beyond Classical Methods and Genetic Algorithm

For the majority of multi-objective decision-making problems, it is almost impossible to obtain Pareto optimal solution set with classical methods. The reason is that most of the multi-objective problems are NP-hard. [6].

Problems with integers are generally quite complicated and have a concave search space and it its hard to find the best solution set. Convergence to Pareto optimal solutions is quite slow in classical methods. Searching for the optimal solutions for these is only possible with heuristic methods [7].

Genetic algorithm (GA) is the application of genetic related mechanisms in optimization problems as an algorithm. Reproduction in genetic generally occurs through mutation and crossover. These are GA operators. The obtained individual (solution) should have a good fitness value to maintain its participation. In this way, strong individuals succeed in survival and can transfer their genes to the next generations. Therefore, better solutions can be achieved by transferring the information on the solutions with good results to the next solutions [8].

To be able to use GA in a computer, the most compatible coding method for the problems should be used. The mostly used methods are binary coding, permutation coding and real number coding.

Selection of the solutions to be used in the next generation is one of the important steps in GA operation. Various selection methods were suggested as a result of research studies. Selection methods commonly used in practice include roulette wheel [9], selection sort [10] and tournament selection [11]. The common feature of these methods is that solutions obtained are probabilistically selected according to their compatibility value. Probability of the solutions with good compatibility value to be included in the next generation is always high.

Crossover and mutation are used to obtain new solutions using the selected solutions. Crossover is more commonly used compared to mutation operator. There are one-point, two-point and multi-point crossover techniques. It is observed that one-point and two-point crossover techniques are used the most. Mutation is an operator applied on solutions obtained after crossover operation. It is applied on the solutions with a small mutation probability identified beforehand and various solutions are obtained in this operation. Mutation operator provides diversity in the formation of next generations. It is possible to obtain bad results as well as good ones with crossover and mutation; however, these solutions are eliminated during selection. In GA, population extent should also be determined in addition to crossover and mutation control parameters. Most of the problems are quite sensitive to these three parameters and the best solution is obtained only with the right selection of parameters [6] .

Considering the studies using GA, Rao and Patel [66] presented a new approach for heat exchangers optimization using modified teaching-learning-based optimization algorithm. They compared the results obtained with GA results. Omkar et al [50] compared the results of the system designed for Vector Evaluated Artificial Bee Colony (VEABC) and its multi-objective optimization with PSO, Artificial Immune System (AIS) and GA.

V. Multi-Objective Genetic Algorithms

There may not be a single best solution for all objectives in multi-objective cases. In this case, the decision maker is asked to select a solution from a compromised finite set. Appropriate solution should perform at an acceptable level for all objectives. It is know that using genetic and evolutionary algorithms for the solution of multi-objective optimization problems is advantageous.

A. *Multi-Objective Genetic Algorithm*

Multi-objective Genetic Algorithm (MOGA) was developed by Fonseca and Fleming [13]. Groups are formed starting with the non-dominated solutions in the first group and solutions dominated for once in the second group. In each group, only the proximity of solutions to each other in the same group is calculated. Following the calculation of proximities, fitness parameter (σ_{share}) which should be a small number depending on the problem is identified. It is easy to calculate fitness values of the solutions in MOGA method. Furthermore, MOGA can be applied in various optimization problems with fitness parameterσ_{share}. However, in this technique, giving the same fitness value to different solutions on the same front may cause the algorithm to look for the solutions on the wrong front. In practice, it has been proved that MOGA is quite sensitive to the type of concerned solution area. Besides, the algorithm does not guarantee that a solution has always a better fitness value than a worse solution [14].

Considering the studies using MOGA, Wang et al [25] optimized the conflicting economic and environmental objectives in environment friendly building designs with MOGA. They presented a case study and demonstrated the effectiveness of their approach for identifying a number of Pareto optimal solutions for green building design. Tahara et al [30] optimized various objective values in ship design with Sequential quadratic programming (SQP) and MOGA. They formulated a multi--objective optimization problem where ship propulsion and maneuverability performances are considered. Pierobon et al [72] realized Rankine cycles optimization for waste heat recovery using MOGA. They observed in the results that they could prevent heat loss.

B. *Non-dominated Sorting Genetic Algorithm*

Non-dominated Sorting Genetic Algorithm (NSGA) was developed by Srivas and Deb [15]. The method starts with sorting the solutions obtained randomly according to their dominance. As in MOGA method, proximity among the solutions in each sub-population is calculated. Solutions are selected with roulette wheel method. In this method, the chance of the solutions in the non-dominated 1st sub-population (P_1) to be selected is higher. New solutions are obtained by applying crossover and mutation on the selected solutions and the algorithm keeps looking for solutions until stopping criterion is formed. NSGA method classifies solutions according to their dominance and assigns fitness values. Various practices have shown that the algorithm is especially σ_{share} sensitive to its parameter [15].

Considering the studies using NSGA, Ahmadi et al [70] aimed to obtain maximized power, thermal efficiency and minimized pressure loss using NSGA in Stirling heat engine design. They observed the results using experimental data.

C. *Elitist Non-dominated Sorting Genetic Algorithm*

Elitist Non-dominated Sorting Genetic Algorithm (NSGA II) was developed by Deb and Goel [18],[19]. This algorithm is similar to NSGA method; however, parameters used in NSGA are not used in NSGA II. NSGA II σ_{share} does not need the fitness parameter. The algorithm gives more chance to the prominent results and it is almost impossible to lose these solutions. Therefore, the algorithm never loses the Pareto optimal solution found until the current step. Solution selection mechanism is used to limit the population size; however, in this case, the algorithm may lose its feature of proximity to the optimal solution. Since the number of solutions in the sub-set F_1 where first non-dominated solutions exist is not higher than the number of main population, all solutions in this set are selected. Therefore, diversity among the selected solutions is not ensured.

In the studies using NSGA-II, Sarkar and Modak [23] developed a solution method for fed-batch bioreactors with NSGA-II. They applied this method on two problems included in the literature and solved with many methods previously. Atashkari et al [26] found out Pareto fronts of conflicting objective functions in the thermodynamic cycle of turbojet engines using ε-elimination and NSGA II. Further, they observed some interesting and important relationships among optimal objective functions and decision variables involved in the thermodynamic cycle of turbojet engines. Deb et al [31] developed a new robustness procedure in order to obtain more realistic results instead of a general pareto front. They used NSGA II to compare global and robust pareto fronts. They developed many limited and unlimited test problems to show the obtained results through simulation. Goel et al [32] realized approximations on the Pareto Front obtained with NSGA II. They showed that the approximated Pareto optimal front (POF) can help visualize balances and quantify values among objectives to select compromise designs. Mandal et al [34] obtained Pareto set by modelling electrical discharge machining (EDM) with NSGA II. They carried out experiments over a wide range of machining conditions for training and verification of the model. Testing results demonstrated that the model is suitable for predicting the response parameters. Jia et al [40] worked on the optimization of energy, cost and scope parameters in the use of wireless network with NSGA II. Numerical and simulation results validate that the procedure to find the optimal balance point among the maximum coverage rate, the least energy consumption, as well as the minimum number of active nodes is fast and effective. Shokri et al [67] suggested a new method by combining NSGA-II and artificial neural networks (ANN) in order to reduce time spent for the solution of multi-objective optimization problems with evolutionary algorithms (EAs). The suggested method was applied on three standard problems and one real-life problem. They were able to considerably reduce the time required to find Pareto optimal front compared to NSGA-II solutions used without ANN. Campomanes-Álvarez et al [74] realized surface simplification optimization for a 3D open model mesh simplification problem. They compared the results obtained from NSGA-II and MOEA/D with the results obtained from two classical methods. Ahmadi et al [78] presented a new approach by using NSGA-II in order to optimize each stage of biomass energy. They showed the Pareto front and observed the results with statistical analyses. Song et al [80] suggested a semi-active battery/supercapacitor (SC) hybrid energy storage system (HESS) for electric vehicles. They realized parameter optimization using NSGA-II in their studies.

D. *Strength Pareto Evolutionary Algorithm*

Strength Pareto Evolutionary Algorithm (SPEA) was developed by Zitzler and Thiele [20]. In this algorithm, diversity in the selection of Pareto optimal solutions is ensured with clustering analysis. Calculation of proximities is easy and additional parameters are not required. In the algorithm, parameter N_t, which is the size of populations where prominent solutions will be tracked, should be identified. Furthermore, balance between the main population size $N$and N_t should be ensured in order to obtain good results from the algorithm. N_t should be neither too big nor too small. If it is too big, the algorithm loses much time with prominent solutions and may not generate other solutions and converge to optimal solutions. If it is too small, solutions in the prominent set are not used enough and it may cause the algorithm to excessively look for the solutions outside the area where optimal solutions exist. A criticised feature of the algorithm is that it gives a fitness value for dominance criterion to the solutions in the main population during the formation of populations. Furthermore, the power values obtained sometimes cannot keep the non-dominated solutions of the same importance at the same value [2].

Considering the studies using SPEA, Farmani et al [24] focused on certain objectives such as capital, operation, life cycle, maintenance costs, system reliability and water quality which should be simultaneously optimized in the design of water systems. Two examples related to design of water systems were applied on each of MOGA, NSGA and SPEA algorithms and their Pareto fronts were compared. Ali et al [57] amended Differential Evolution (DE) algorithm and transformed it into Multi-Objective Differential Evolution Algorithm (MODEA). They introduced new approaches to mutation and selection mechanisms in the meanwhile. The new approach obtained was applied on different multi-objective problems and the results were compared with algorithms NSGA-II, SPEA and Pareto Archived Evolutionary Strategy (PAES) developed by Knowles ve Corne [21].

E. *Strength Pareto Evolutionary Algorithm 2*

Strength Pareto Evolutionary Algorithm 2 (SPEA2) was developed by Zitzler et al. [22]. It is the developed version of SPEA method. SPEA2 introduces a better scoring mechanism, an intensity estimation technique and a developed archive (secondary community) management compared to SPEA method. SPEA2 uses fine-grained fitness assignment strategy using intensity information. Furthermore, size of the archive externally storing the non-dominated individuals is stable. If the number of non-dominated individuals is lower than the previously identified archive size, the archive is filled with non-dominated individuals. Additionally; the clustering technique used when non-dominated surface is over the archive size was replaced with an alternative downsizing method that has similar features but does not lose front points. Finally, another difference from SPEA is that only the archive members are used in selection process.

Considering the studies using SPEA-II, Durillo and Nebro [54] designed a developed optimization practice with NSGA-II and SPEA2. In this way, they realized meta-heuristic and experimental practices in the solution of multi-objective optimization applications. Xue et al [75] realized a feature selection optimization using PSO with the aim of reducing surplus and unnecessary features in the data sets and obtain more accurate results in a shorter time in classification. They compared the results obtained with NSGA-II, SPEA-II and PAES.

VI. Particle Swarm Optimization

Particle Swarm Optimization is a swarm-based algorithm developed by Kennedy and Eberhart in 1995 with the inspiration from fish schools and bird flocks in the nature [12]. The algorithm was built on the behaviours of flock animals practices in order to meet essential needs. Particle concept used in PSO algorithm represents each individual in the swarm. Each individual in the swarm has position information indicating the position of the individual in d-size solution space and speed information indicating its movement in d-size solution space. PSO is a repetitive algorithm. Therefore, speed and position information of the particle is updated during each repetition. While speed information of the particle is updated, not only the heuristic speed information from the previous step but also cognitive and social experiences are used. For the updated position information of the particle, position information and updated speed information of the particle from the previous step are used. [12, 16].

Considering the studies using PSO; Wang and Singh [35] suggested fuzzified multi-objective particle swarm optimization (FMOPSO) for the compromising economic and environmental objectives in electricity transmission. The performance of the suggested approach was compared with Weighted Clustering (WA) and evolutionary multi-objective optimization algorithms. Tripathi et al [36] developed Time Variant Multi-Objective Particle Swarm Optimization (TV-MOPSO) procedure based on PSO multi objective optimization approaches and measured the performance by comparing with algorithms such as NSGA II and PESA II. Zhang and Liu [39] presented a new formulation with fuzzy adaptive PSO (FAPSO) for multi-objective reactive power and voltage control. The proposed approach has been examined and tested with promising numerical results of the IEEE 30-bus and IEEE 118-bus power systems. Zhang and Xing [47] developed a construction method with PSO for the time–cost–quality trade-off problem. They applied the suggested method with computerized analyses and verified. Moslehi and Mahnam [49] presented a new approach for flexible job-shop scheduling problem using particle swarm optimization and local search. They compared the efficiency of the approach presented with other algorithms in the literature. Qasem and Shamsuddin [51] applied TV-MOPSO on radial basis function (RBF) used in the diagnosis of medical disorders. They compared the results with MOPSO and NSGA II. Yildiz and Solanki [59] presented a new method for the multi-objective optimization of crashworthiness of vehicles. The PSO-based method was applied on two optimization problems in the literature. Khalili-Damghani et al [64] suggested dynamic self-adaptive multi-objective particles warm optimization (DSAMOPSO) for the solution of multi-objective reliability redundancy allocation problems (MORAPs). They compared the results obtained from test problems with TV-MOPSO and NSGA-II results. Garg and Sharma [65] reformulated multi-objective reliability-redundancy allocation problem with fuzzy multi-objective optimization problem (FMOOP). They solved the fuzzy MOOP obtained using PSO. The approach has been demonstrated through the case study of a pharmaceutical plant situated in the northern part of India. Zhang et al [68] realized an uncertain orbit optimization using risk and orbit distance parameters in the problems related to orbit planning for robots. In their studies, they used PSO and observed the results from simulation. Taormina and Chau [82] created a multi-objective problem using cross-validation for PSO-trained neural network river forecasting (NNRF). They solved this problem with multi objective fully informed particle swarm (MOFIPS) approach using actual data.

VII. Other Methods and New Approaches

New approaches presented for the solution of MOOPs and methods developed for special systems are handled.

Mahapatra and Roy [27] developed a new fuzzy multi-objective optimization method in order to increase system reliability. The main aim was to increase reliability while reducing the cost. Huang et al [28] developed a special interactive fuzzy multi-objective optimization model for engineering designs. In this model, balance matrix was used and weight coefficients of objective functions were identified depending on preference. Igel et al [33] transformed covariance matrix adaptation evolution strategy (CMA-ES) delivering quite good results in single-objective optimization for multi-objective (MO-CMA-ES) and elitists problems. Obtained results were compared with NSGA II. Jaeggi et al [37] adapted Tabu Search algorithm for the solution of MOOPs. All results were compared with MOTS (Multi-Objective Tabu Search), PRMOTS (Path Relinking Multi-Objective Tabu Search) and NSGA II. Thiele et al [41] elaborated the obtained pareto optimal set with the instantaneous preferences of the user in each repetition using multi-objective evolutionary algorithm (MOEA). In this way, they managed to include ignored but essential solutions in the pareto optimal set. Kim et al [42] designed multi-objective population-based incremental learning (MOPBIL) for the fuzzy path planning of robots. Simulation and experiment results show the effectiveness of the proposed MOPBIL from the viewpoint of the proximity to the Pareto-optimal set, size of the dominated space, coverage of two sets and diversity metric. Chaudhuri and Deb [43] developed a procedure to assist users to select a specific solution on the obtained pareto optimal set with multi-objective evolutionary optimization (EMO). Huang et al [52] developed a new online multi-objective optimization algorithm for membrane computing. They prevented the formation of undesired and wrong solutions with online controls in this dynamic algorithm. Yang [55] developed the multi-objective bat algorithm (MOBA). He applied the algorithm on the problems related to welded beam design and observed its success in the results of simulation. Fesanghary et al [58] presented a model based on harmony search algorithm (HS) in order to minimize life cycle cost (LCC) and carbon dioxide equivalent (CO2-eq) and maximize energy efficiency of the buildings. To measure the efficiency of the proposed approach, they tested the performance of the model on a typical single- family house. Jiménez et al [69] conducted an optimization study on production planning using fuzzy multi-objective evolutionary algorithm. They compared the results with NSGA II. Sindhya et al [71] attempted to eliminate the problems arising in optimization process with multi-objective evolutionary algorithms (MOEA) using new hybrid systems. NSGA-II and MOEA types were transformed into hybrid systems and the results were compared. Niu et al [73] developed a new multi-objective Bacterial Foraging Optimization (MBFO) algorithm and compared the obtained results with NSGA-II and MOPSO. Wang et al [76] presented an approach for cost-energy optimization in the coalfired thermal power plants. In their approach, they used multi-objective differential evolution (MODE) and showed the results with simulation. Huang [77] developed a design optimization for design exploration of three-dimensional transverse jet in a supersonic crossflow and observed the results with simulation. Wang et al [81] presented a multi-objective optimization for combined cooling, heating and power system (CCHP). In the experiments conducted, it was found out that a more comprehensive solution set is required to obtain more successful results from optimization.

VIII. CONCLUSIONS

In this study, 61 articles published in the last 15 years are selected and analysed. Each article is handled under separate headings in terms of their subjects, methodology and results. As a results of the analysis, it is observed that operations using multi-objective optimization techniques are quite successful in the solution of comprehensive problems which is a huge challenge for operators in the special areas.

The method used in the solution of MOOPs should be chosen considering the objectives and constraints peculiar to each area. Because classical and heuristic methods do not deliver the same results in the same operation. While determining the classical and heuristic methods to be used in article analyses, the extent of information available on the handled problem is quite effective. If the importance of each objective and constraint is know in numbers for the problem aimed to be solved, mostly classical methods are preferred. To be more precise, the more the problem is complicated and unknown, the more the probability of selecting heuristic methods increases. Classical methods are chosen when complete and certain results are desired to be obtained. In the operations where solution performance is low due to the complexity of the problem, pareto optimal set which can satisfy the operator and show the approximate results is obtained. In the articles, finding the algorithms that can present the pareto optimal set most accurately and in the shortest notice becomes prominent as the main aim.

Another outstanding subject in the articles is NSGA-II which is one of the multi-objective genetic algorithm types among heuristic methods. It was observed this algorithm delivers the most diversity in the solution of problems. This algorithm either serves as the main method of many operations or the developers use a variant of this algorithm. The abundance of NSGA-II use in the measurement of special algorithms developed is outstanding. In the studies using PSO, the most remarkable feature of PSO is its easy practice in most of the systems. Developers have experimented many variants of PSO on the same problem and compared the results.

Considering the studies in the special areas, it is observed that multi-objective optimization methods in the literature are either used in the same way or their variants are applied. The success of algorithms developed in the studies are measured with the comparison of results from many methods applied on the same problem.

One of the biggest problems arisen during the studies is the ignorance of essential solutions for users while finding the pareto optimal front. All these observations bring this question to the mind: Is it possible to develop a multi-objective optimization method that can be applied on optimization problems in each area and provide the maximum performance in the solution? For the answer of this question, it is observed that studies on the performance of multi-objective optimization among the articles are quite important. The outstanding point in these studies is the interaction between application and user. If the future studies focus on increasing the performance and robustness of multi-objective optimization, methods delivering results where most approximate values to the solution are found in the shortest notice and essential solutions for the operators are not ignored can be developed.

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