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Research Article

New Exponential and Complex Traveling Wave Solutions to the Konopelchenko-Dubrovsky Model

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The Konopelchenko-Dubrovsky (KD) system is presented by the application of the improved Bernoulli subequation function method (IBSEFM). First, The KD system being Nonlinear partial differential equations system is transformed into nonlinear ordinary differential equation by using a wave transformation. Last, the resulting equation is successfully explored for new explicit exact solutions including singular soliton, kink, and periodic wave solutions. All the obtained solutions in this study satisfy the Konopelchenko-Dubrovsky model. Under suitable choice of the parameter values, interesting two- and three-dimensional graphs of all the obtained solutions are plotted.

1. Introduction

Various complex nonlinear phenomena in different fields of nonlinear sciences such as fluid mechanic, plasma physics, and optical fibers can be expressed in the form of nonlinear partial differential equations (NPDEs). Many analytical methods for solving such type of equations have been developed and used by different researchers from all over the world. Zayed et al. [1] used the generalized Kudryashov in addressing some NPDEs arising in mathematical physics. Wang [2] investigated the Sharma-Tosso-Olver by using the extended hamoclinic test function method. Nofal [3] solved some nonlinear partial differential equation by using the simple equation method. Shang [4] obtained the exact solutions of the long-short wave resonance equation by using the extended hyperbolic function method. Wazzan [5] used the modified tanh-coth method in solving the generalized Burgers-fisher and Kuramoto-Sivashinsky equations. Ren et al. [6] applied the generalized algebra method to the (2+1)-dimensional Boiti-Leon-Pempinelli system. In general various studies in this context have been submitted to the literature [7–23].

In this paper, the Konopelchenko-Dubrovsky (KD) equations [24] are investigated by using the improved Bernoulli subequation function method [25, 26].

The Konopelchenko-Dubrovsky (KD) equations are given by [24]

$$\begin{aligned} u_t - u_{xxx} - 6buu_x + \frac{3}{2}a^2u^2u_x - 3v_y + 3av_xu = 0 \\ u_y = v_x, \end{aligned} \tag{1}$$

where $u = u(x, y, t)$, $v = v(x, y, t)$ and a, b are constants.

Various analytical approaches have been used in obtaining the exact solutions to the Konopelchenko-Dubrovsky equations. Sheng [27] used the improved F-expansion method in addressing (1), Wazwaz [28] employed the tanh-sech method, the cosh-sinh method, and the exponential functions method for obtaining the analytical solutions to (1), and Kumar et al. [29] solved the KD equations by traveling wave hypothesis and lie symmetry approach. Song and Zhang [30] utilized the extended Riccati equation rational expansion method and Feng and Lin [31] applied the improved Riccati mapping approach. Bekir [32] used the extended tanh method. Xia et al. [33] employed the new modified extended tanh function method.

2. The IBSEFM

In this section, we present Improved Bernoulli subequation function method (IBSEFM) formed by modifying the

Bernoulli subequation function method. Therefore, we consider the following four steps.

Step 1. Let us consider the nonlinear partial differential equation given as

$$P\left(u, u_x, u_t, u_{xt}, \cdots\right) = 0 \quad (2)$$

and the wave transformation

$$u(x,t) = U(\eta), \quad \eta = kx - ct, \quad (3)$$

Substituting (3) into (2) gives the following nonlinear ordinary differential equation:

$$N\left(U, U', U'', U''', \cdots\right) = 0. \quad (4)$$

Step 2. The trial solution of (4) is assumed to be

$$U(\eta) = \frac{\sum_{i=0}^{n} a_i F^i(\eta)}{\sum_{j=0}^{m} b_j F^j(\eta)} = \frac{a_0 + a_1 F(\eta) + a_2 F^2(\eta) + \cdots + a_n F^n(\eta)}{b_0 + b_1 F(\eta) + b_2 F^2(\eta) + \cdots + b_m F^m(\eta)}. \quad (5)$$

According to the Bernoulli theory, we have the general form of the Bernoulli differential equation as

$$F' = wF + dF^M, \quad w \neq 0,\ d \neq 0,\ M \in R - \{0, 1, 2\}, \quad (6)$$

where $F = F(\eta)$ is Bernoulli differential polynomial. Substituting (6) into (4) gives equations in degrees of F as

$$\Omega(F) = \rho_s F^s + \cdots + \rho_1 F + \rho_0 = 0. \quad (7)$$

The values of n, m, and M are to be determined by using the homogeneous balance principle.

Step 3. Setting the summation of the coefficients of $\Omega(F)$ and equating each summation to zero yields an algebraic system of equation.

$$\rho_i = 0, \quad i = 0, \cdots, s. \quad (8)$$

Solving this system of equation, we reach the values of $a_0, \cdots, a_n$ and $b_0, \cdots, b_m$.

Step 4. Equation (6) has the following solutions depending on the values of b and d:

$$F(\eta) = \left[\frac{-d}{w} + \frac{E}{e^{w(M-1)\eta}}\right]^{1/(1-M)}, \quad w \neq d, \quad (9)$$

$$F(\eta) = \left[\frac{(E-1) + (E+1)\tanh\left(w(1-M)\eta/2\right)}{1 - \tanh\left(w(1-M)\eta/2\right)}\right]^{1/(1-M)}, \quad w = d, \quad (10)$$

Using a complete discrimination system for the polynomial of F, we obtain the analytical solutions to (4) with the help of Wolfram Mathematica. For a better interpretation of results obtained in this way, we can plot two- and three-dimensional figures of analytical solutions by considering suitable values of parameters.

3. Application

In this section, we present the application of IBSEFM method to the Konopelchenko-Dubrovsky (KD) equations. Using the wave transformation on (1)

$$\begin{aligned} u(x,y,t) &= U(\gamma), \quad \gamma = x + my - ct, \\ v(x,y,t) &= V(\gamma), \quad \gamma = x + my - ct, \end{aligned} \quad (11)$$

we get the following system of nonlinear ordinary differential equations:

$$\begin{aligned} -cU' - U''' - 6bUU' + \frac{3}{2}a^2U^2U' - 3V' + 3aV'U &= 0, \\ mU' &= V'. \end{aligned} \quad (12)$$

Integrating the second equation in the system (12), we get

$$mU = V. \quad (13)$$

Inserting (13) into the first equation of (12), we get the following single nonlinear ordinary differential equation:

$$(6ma - 12b)UU' - \left(2c + 6m^2\right)U' - 2U''' + 3a^2U^2U' = 0. \quad (14)$$

Finally, integrating (14), we have

$$(3ma - 6b)U^2 - \left(2c + 6m^2\right)U - 2U'' + a^2U^3 = 0. \quad (15)$$

Balancing (15) by considering the highest derivative and the highest power, we obtain $m + M = n + 1$.

Choosing $M = 3$ and $m = 1$, gives $n = 3$. Thus, the trial solution to (1) takes the following form:

$$U(\gamma) = \frac{a_0 + a_1F(\gamma) + a_2F^2(\gamma) + a_3F^3(\gamma)}{b_0 + b_1F(\gamma)}, \quad (16)$$

where $F' = wF + dF^3$, $w \neq 0$, $d \neq 0$. Substituting (16), its second derivative along with $F' = wF + dF^3$, $w \neq 0$, $d \neq 0$ into (15) yields a polynomial in F. We collect a system of algebraic equations from the polynomial by equating each summation of the coefficients of F which have the same power. Solving the system of the algebraic equations yields the values of the parameter involved. Substituting the obtained values of the parameters into (16), yields the solutions to (1).

Case 1.

$$
\begin{aligned}
a_0 &= -\frac{2(-2W + m\alpha) b_0}{\alpha^2}, \\
a_3 &= \frac{\alpha^2 a_1 a_2}{4Wb_0 - 2m\alpha b_0}, \\
b_1 &= \frac{\alpha^2 a_1}{4W - 2m\alpha}, \\
c &= -\frac{4\left(W^2 - mW\alpha + m^2\alpha^2\right)}{\alpha^2}, \\
\sigma &= -\frac{m}{2} + \frac{W}{\alpha}, \\
d &= \frac{\alpha a_2}{4b_0}.
\end{aligned} \tag{17}
$$

Case 2.

$$
\begin{aligned}
a_0 &= \frac{a_1 b_0}{b_1}, \\
a_2 &= \frac{d a_1 b_0}{\sigma b_1}, \\
a_3 &= \frac{d a_1}{\sigma}, \\
c &= -3m^2 - 4\sigma^2, \\
\alpha &= -\frac{4\sigma b_1}{a_1}, \\
W &= -\frac{2(m - 2\sigma)\sigma b_1}{a_1}.
\end{aligned} \tag{18}
$$

Case 3.

$$
\begin{aligned}
a_0 &= \frac{a_1 b_0}{b_1}, \\
a_2 &= -\frac{4 d b_0}{\alpha}, \\
a_3 &= -\frac{4 d b_1}{\alpha}, \\
c &= -3m^2 + \frac{\alpha^2 a_1^2}{2b_1^2}, \\
W &= \frac{m\alpha}{2}, \\
\sigma &= -\frac{\alpha a_1}{2b_1}.
\end{aligned} \tag{19}
$$

Case 4.

$$
\begin{aligned}
a_0 &= \frac{a_1 b_0}{b_1}, \\
a_2 &= -\frac{2\sqrt{2} d a_1 b_0}{\sqrt{(c + 3m^2) b_1^2}}, \\
a_3 &= -\frac{2\sqrt{2} d a_1 b_1}{\sqrt{(c + 3m^2) b_1^2}}, \\
\alpha &= -\frac{\sqrt{2}\sqrt{(c + 3m^2) b_1^2}}{a_1}, \\
W &= -\frac{m\sqrt{(c + 3m^2) b_1^2}}{\sqrt{2} a_1}, \\
\sigma &= -\frac{\sqrt{(c + 3m^2) b_1^2}}{\sqrt{2} b_1}.
\end{aligned} \tag{20}
$$

Case 5.

$$
\begin{aligned}
a_0 &= \frac{a_1 b_0}{b_1}, \\
a_2 &= -\frac{2 i d a_1 b_0}{\sqrt{c + 3m^2} b_1}, \\
a_3 &= -\frac{2 i d a_1}{\sqrt{c + 3m^2}}, \\
\alpha &= \frac{2 i \sqrt{c + 3m^2} b_1}{a_1}, \\
W &= -\frac{\left(c + m\left(3m - i\sqrt{c + 3m^2}\right)\right) b_1}{a_1}, \\
\sigma &= \frac{1}{2} i \sqrt{c + 3m^2}.
\end{aligned} \tag{21}
$$

Substituting (17) into (16), gives

$$
u_1(x, y, t) = \frac{4W - 2m\alpha}{\alpha^2} + \frac{a_2}{\left(e^{-2(-ct+x+my)\sigma}\epsilon - d/\sigma\right) b_0}. \tag{22}
$$

Substituting (18) into (16), gives

$$
u_2(x, y, t) = \frac{\epsilon\sigma a_1}{\left(-d e^{2(-ct+x+my)\sigma} + \epsilon\sigma\right) b_1}. \tag{23}
$$

Substituting (19) into (16), gives

$$
u_3(x, y, t) = \frac{4\sigma\left(1 + \epsilon\sigma / \left(d e^{2(-ct+x+my)\sigma} - \epsilon\sigma\right)\right)}{\alpha} + \frac{a_1}{b_1}. \tag{24}
$$

Substituting (20) into (16), gives

$$u_4(x,y,t) = \frac{\left(-2d^2 + e^{2\sqrt{2}(-ct+x+my)\sqrt{(c+3m^2)b_1^2}/b_1}\left(c+3m^2\right)\epsilon^2\right)a_1\sqrt{(c+3m^2)b_1^2}}{\left(e^{\sqrt{2}(-ct+x+my)\sqrt{(c+3m^2)b_1^2}/b_1}(c+3m^2)\epsilon b_1 + \sqrt{2}d\sqrt{(c+3m^2)b_1^2}\right)\left(\sqrt{2}db_1 + e^{\sqrt{2}(-ct+x+my)\sqrt{(c+3m^2)b_1^2}/b_1}\epsilon\sqrt{(c+3m^2)b_1^2}\right)}. \quad (25)$$

Substituting (21) into (16), gives

$$u_5(x,y,t) = \frac{\sqrt{c+3m^2}\epsilon a_1}{\left(2ide^{i\sqrt{c+3m^2}(-ct+x+my)} + \sqrt{c+3m^2}\epsilon\right)b_1}. \quad (26)$$

4. Results and Discussion

This study uses the IBSEFM to obtain some travelling wave solutions to the Konopelchenko-Dubrovsky equations such as the singular soliton, kink-type soliton, and the periodic wave solutions. The results are presented in exponential function structure in Section 3. When the basic relation $e^x = \sinh(x) + \cosh(x)$ is used, solutions (22)-(26) become

$$u_1(x,y,t) = \frac{4W}{\alpha^2} - \frac{2m}{\alpha} + b_0a_2\left(-\frac{d}{\sigma} + \epsilon\cosh\left[2ct\sigma - 2x\sigma - 2my\sigma\right] + \epsilon\sinh\left[2ct\sigma - 2x\sigma - 2my\sigma\right]\right)^{-1}, \quad (27)$$

$$u_2(x,y,t) = \epsilon\sigma a_1b_1\left(\epsilon\sigma - d\cosh\left[2ct\sigma - 2x\sigma - 2my\sigma\right] + d\sinh\left[2ct\sigma - 2x\sigma - 2my\sigma\right]\right)^{-1}, \quad (28)$$

$$u_3(x,y,t) = \frac{4\sigma}{\alpha} + \frac{4\epsilon\sigma^2}{\alpha\left(-\epsilon\sigma + d\cosh\left[2ct\sigma - 2x\sigma - 2my\sigma\right] - d\sinh\left[2ct\sigma - 2x\sigma - 2my\sigma\right]\right)} + \frac{a_1}{b_1}, \quad (29)$$

$$\begin{aligned}
u_4(x,y,t) = {} & \Bigg(\Bigg(-2d^2 \\
& + \left(c+3m^2\right)\epsilon^2\left(\cosh\left[\frac{2\sqrt{2}(-ct+x+my)\sqrt{cb_1^2+3m^2b_1^2}}{b_1}\right] + \sinh\left[\frac{2\sqrt{2}(-ct+x+my)\sqrt{cb_1^2+3m^2b_1^2}}{b_1}\right]\right)\Bigg) \\
& \cdot a_1\sqrt{cb_1^2+3m^2b_1^2}\Bigg) \\
& \times\Bigg(\Bigg(\left(c+3m^2\right)\epsilon\left(\cosh\left[\frac{\sqrt{2}(-ct+x+my)\sqrt{cb_1^2+3m^2b_1^2}}{b_1}\right] + \sinh\left[\frac{\sqrt{2}(-ct+x+my)\sqrt{cb_1^2+3m^2b_1^2}}{b_1}\right]\right)b_1 \\
& + \sqrt{2}d\sqrt{cb_1^2+3m^2b_1^2}\Bigg)\Bigg(\sqrt{2}db_1 + \epsilon\cosh\left[\frac{\sqrt{2}(-ct+x+my)\sqrt{cb_1^2+3m^2b_1^2}}{b_1}\right]\sqrt{cb_1^2+3m^2b_1^2} \\
& + \epsilon\sinh\left[\frac{\sqrt{2}(-ct+x+my)\sqrt{cb_1^2+3m^2b_1^2}}{b_1}\right]\sqrt{cb_1^2+3m^2b_1^2}\Bigg)\Bigg)\Bigg)^{-1},
\end{aligned} \quad (30)$$

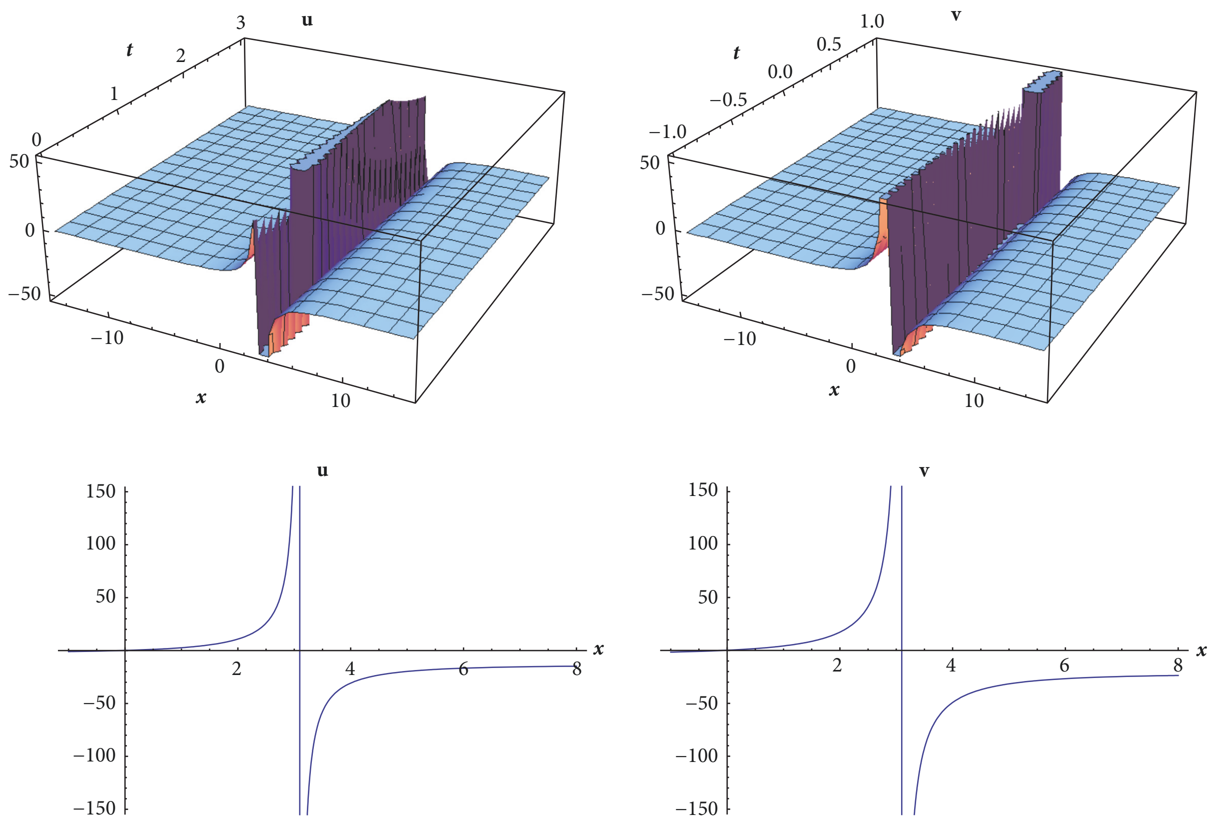

FIGURE 1: The 3D and 2D surfaces of the solution (22) by considering the values $b_0 = 0.5$, $a_2 = 0.2$, $k = 1.6$, $W = 0.3$, $c = 0.2$, $d = 0.01$, $\propto = 0.8$, $\sigma = 0.3$, $\epsilon = 0.9$, $y = 1.5$, and $t = 0.007$ *for 2D*.

where $cb_1^2 + 3m^2b_1^2 > 0$ for valid solution,

$$u_5(x, y, t) = \sqrt{c + 3m^2}\epsilon a_1 b_1 (\sqrt{c + 3m^2}\epsilon + 2id \cos\left[\sqrt{c + 3m^2}(-ct + x + my)\right] - 2d \sin\left[\sqrt{c + 3m^2}(-ct + x + my)\right], \tag{31}$$

where $c + 3m^2 > 0$ for valid solution.

It has been reported in the literature that some computational approaches have been utilized to obtain the solutions of the Konopelchenko-Dubrovsky equations. Sheng [27] reported that some Jacobi elliptic function solutions, soliton-like solutions, trigonometric function solutions improved F-expansion method. Wazwaz [28] obtained some solitary wave, periodic wave, and kink solutions to this equation by using the tanh–sech method and the cosh–sinh scheme. The traveling wave hypothesis and the lie group approach were used on the Konopelchenko-Dubrovsky equations by Kumar et al. [29] and some solitary wave, periodic singular waves, and cnoidal and snoidal solutions were reported. Song and Zhang [30] employed the extended Riccati equation rational expansion method to (1) and obtained some rational formal hyperbolic function solutions, rational formal triangular periodic solutions, and rational solutions. Via the improved mapping approach and variable separation method, Feng and Lin [31] constructed some solitary wave solutions, periodic wave solutions, and rational function solutions. Bekir [32] constructed some kink and singular solitons by using the extended tanh method. Xia et al. [33] employed the modified extended tanh function method to the Konopelchenko-Dubrovsky equation, and soliton-like solutions, periodic formal solution, and rational function solutions were successfully constructed. We observed that the analytical method use in this study revealed the wave solutions in exponential function structure that can be converted to solutions with hyperbolic and trigonometric function structure.

Remark 1. Solutions (22), (23), and (24) are singular soliton solutions. Solution (25) is a kink-type soliton and solution (26) is a periodic wave solution.

5. Conclusions

In this paper, the IBSEFM is applied to the KD equations. We successfully obtained some new traveling wave solutions to the studied model such as complex and exponential function solutions. We presented the 2D and 3D graphs to each of the obtained solutions with the help of some powerful software program.

It has been observed that these solutions have verified nonlinear KD equations. These traveling wave solutions, Figures 1, 2, 3, 4, 5, and 6, have been formed. Moreover, if we chose $M = 3$, $m = 2$, and $n = 4$, we could obtain some other

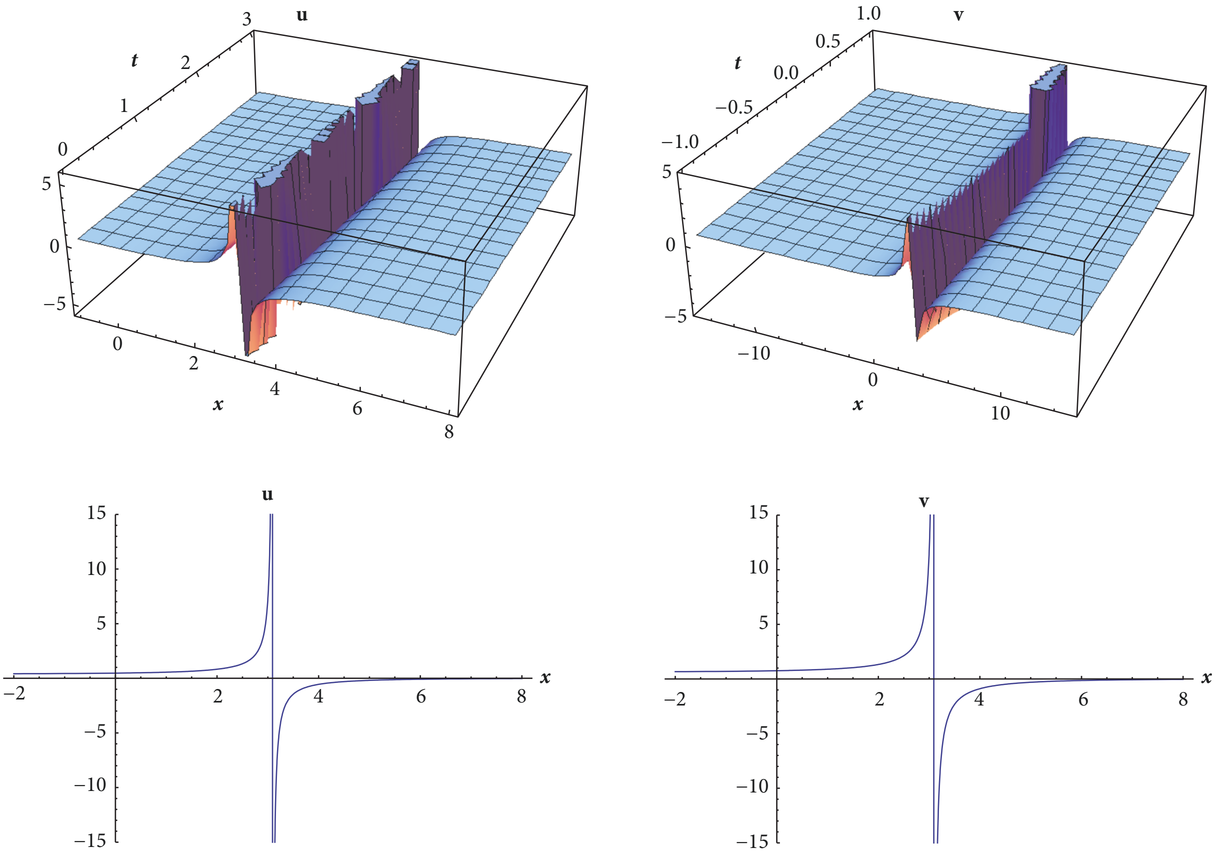

FIGURE 2: The 3D and 2D surfaces of the solution (23) by considering the values $b_1 = 0.5$, $a_1 = 0.2$, $k = 1.6$, $c = 0.2$, $d = 0.01$, $\sigma = 0.3$, $\epsilon = 0.9$, $y = 1.5$, and $t = 0.007$ *for 2D*.

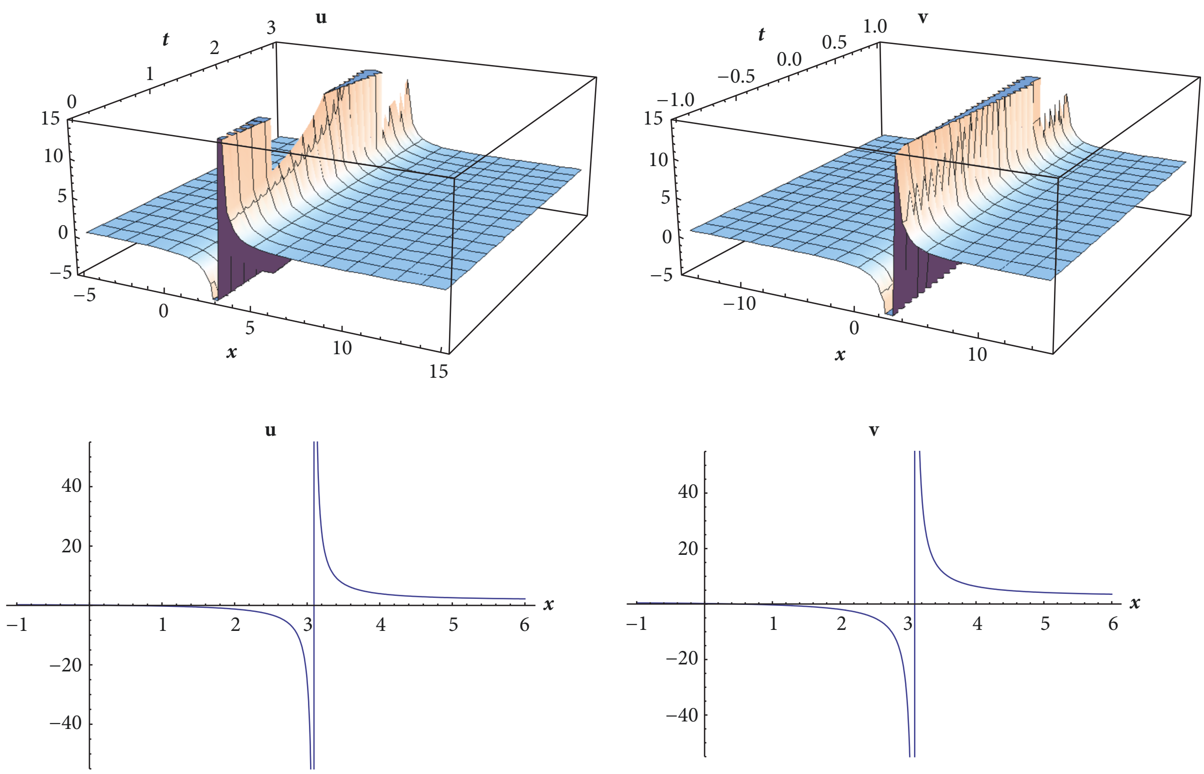

FIGURE 3: The 3D and 2D surfaces of the solution (24) by considering the values $b_1 = 0.5$, $a_1 = 0.2$, $k = 1.6$, $c = 0.2$, $d = 0.01$, $\propto = 0.8$, $\sigma = 0.3$, $\epsilon = 0.9$, $y = 1.5$, and $t = 0.007$ *for 2D*.

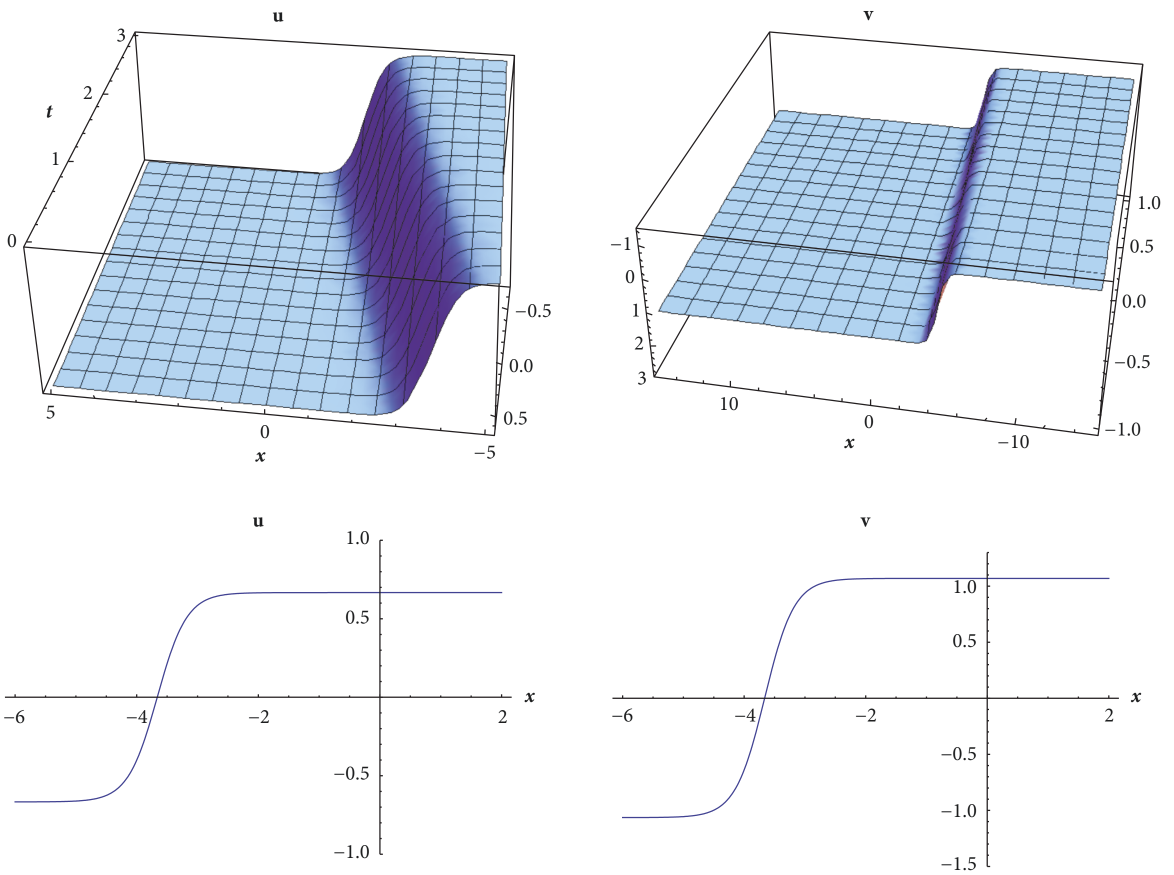

Figure 4: The 3D and 2D surfaces of the solution (25) by considering the values $b_1 = 0.3$, $a_1 = 0.2$, $k = 1.6$, $c = 0.8$, $d = 0.01$, $y = 1.5$, $\epsilon = 0.9$, and $t = 0.007$ *for 2D*.

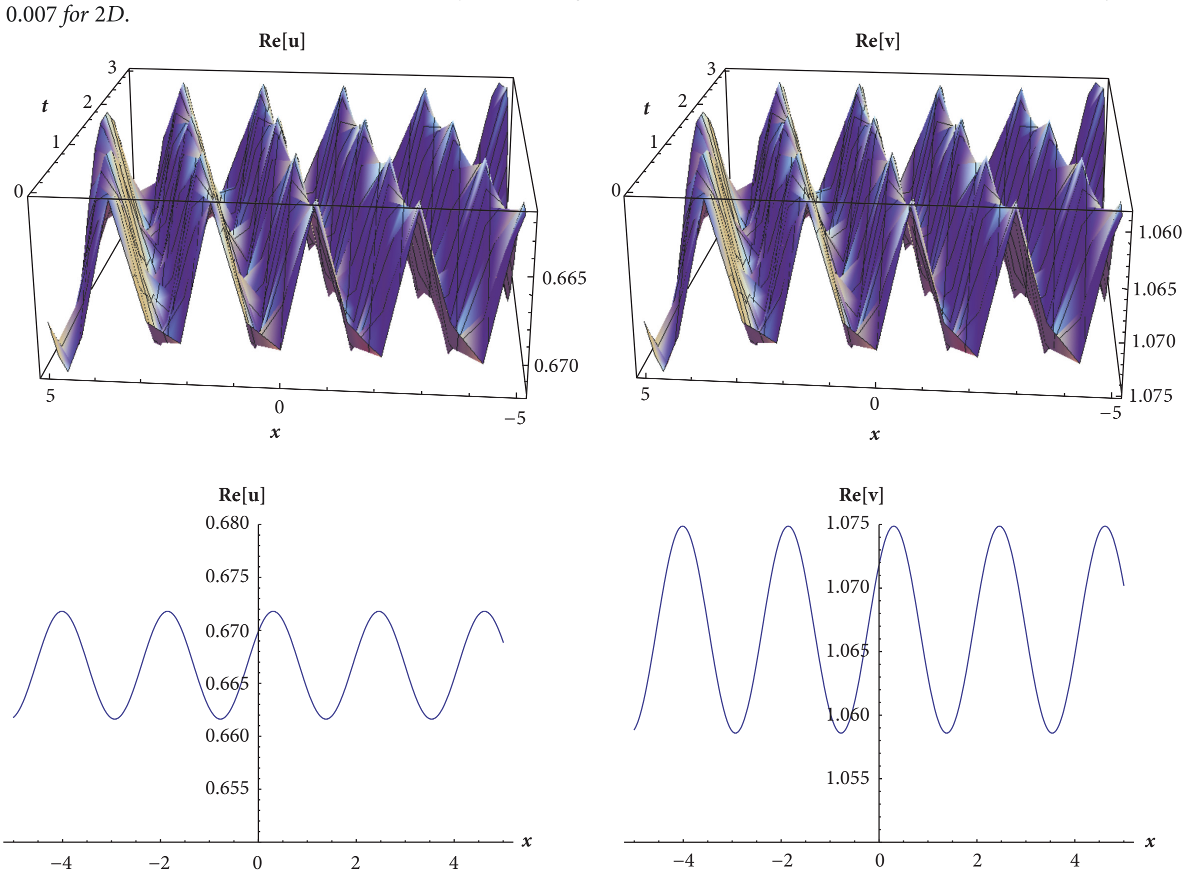

Figure 5: The 3D and 2D surfaces of the analytical solution (26) (real part) by considering the values $b_1 = 0.3$, $a_1 = 0.2$, $k = 1.6$, $c = 0.8$, $d = 0.01$, $y = 1.5$, $\epsilon = 0.9$, and $t = 0.007$ *for 2D*.

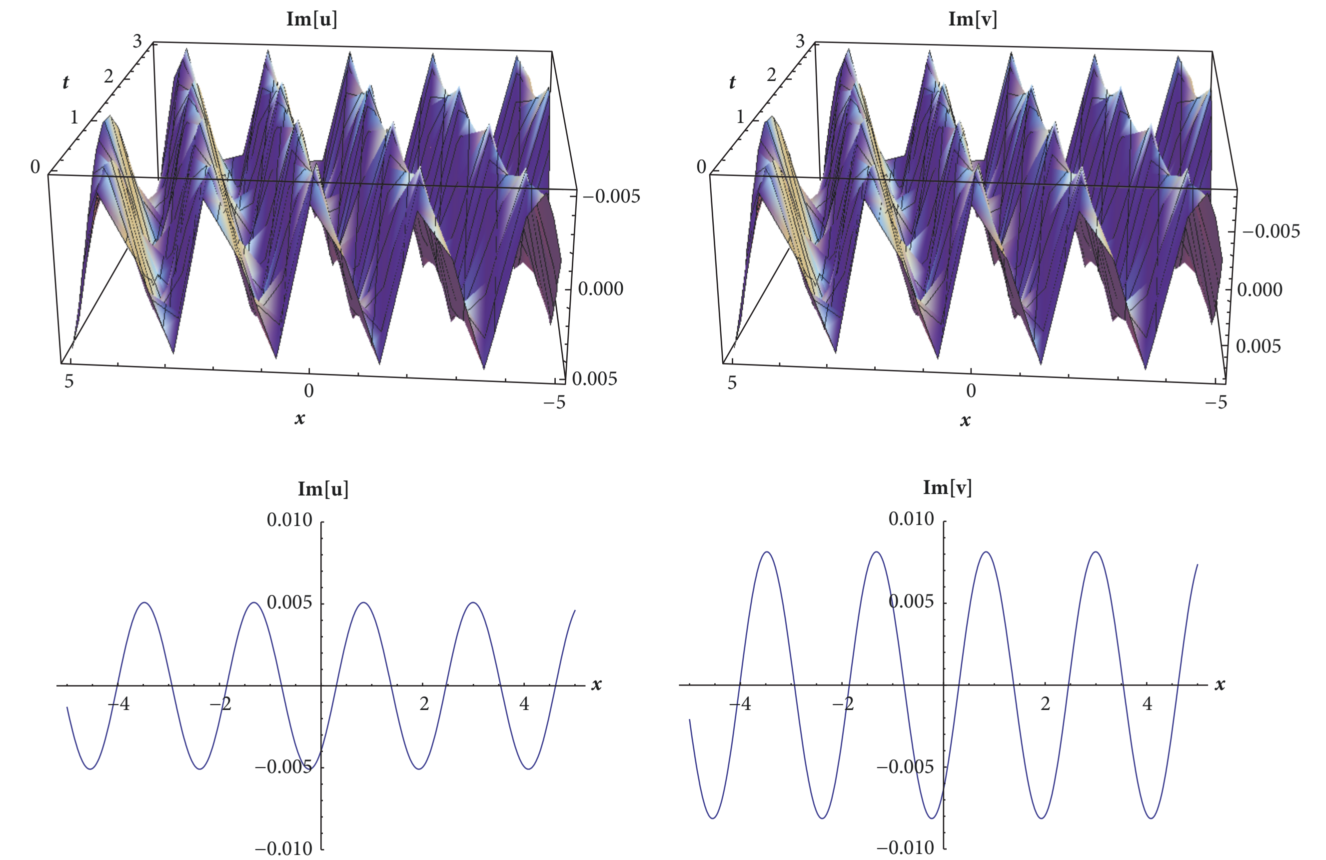

Figure 6: The 3D and 2D surfaces of the analytical solution (26) (imaginer part) by considering the values $b_1 = 0.3$, $a_1 = 0.2$, $k = 1.6$, $c = 0.8$, $d = 0.01$, $y = 1.5$, $\epsilon = 0.9$, and $t = 0.007$ *for 2D*.

new traveling wave solutions to the nonlinear KD equations. When we compare our results with some of the existing results in the literature, we observe that the results reported in this study especially (25) and (26) are new complex and exponential solutions. All the reported solutions in this study verify the Konopelchenko-Dubrovsky equation. Our results might be useful in explaining the physical meaning of various nonlinear models arising in the field of nonlinear sciences. IBSEFM is powerful and efficient mathematical tool that can be used to handle various nonlinear mathematical models.

The performance of this method is effective and reliable. In our research, these traveling wave solutions have not submitted to the literature in advance.

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

Part of this article is presented in 3rd International Conference on Computational Mathematics and Engineering Sciences (CMES 2018).

Conflicts of Interest

The author declares that he has no conflicts of interest.

Authors' Contributions

The author read and approved the final manuscript.

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References

[1] E. M. E. Zayed, G. M. Moatimid, and A. G. Al-Nowehy, "The generalized kudryashov method and its applications for solving nonlinear PDEs in mathematical physics," *Scientific Journal of Mathematics Research*, vol. 5, no. 2, pp. 19–39, 2015.

[2] C. Wang, "Dynamic behavior of traveling waves for the Sharma-Tasso-Olver equation," *Nonlinear Dynamics*, vol. 85, no. 2, pp. 1119–1126, 2016.

[3] T. A. Nofal, "Simple equation method for nonlinear partial differential equations and its applications," *Journal of the Egyptian Mathematical Society*, vol. 24, no. 2, pp. 204–209, 2016.

[4] Y. Shang, "The extended hyperbolic function method and exact solutions of the long-short wave resonance equations," *Chaos, Solitons and Fractals*, vol. 36, no. 3, pp. 762–771, 2008.

[5] L. Wazzan, "A modified tanh-coth method for solving the general Burgers-Fisher and the Kuramoto-Sivashinsky equations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 6, pp. 2642–2652, 2009.

[6] Y.-J. Ren, S.-T. Liu, and H.-Q. Zhang, "A new generalized algebra method and its application in the (2+1)-dimensional boiti-leon-pempinelli equation," *Chaos, Solitons and Fractals*, vol. 32, no. 5, pp. 1655–1665, 2007.

[7] C. Cattani, "Harmonic wavelet solutions of the Schrodinger equation," *International Journal of Fluid Mechanics Research*, vol. 30, no. 5, pp. 463–472, 2003.

[8] C . Cattani and Y. Y. Rushchitskii, "Cubically nonlinear elastic waves: wave equations and methods of analysis," *International Applied Mechanics*, vol. 39, no. 10, pp. 1115–1145, 2003.

[9] A.-M. Wazwaz and M. S. Mehanna, "A variety of exact travelling wave solutions for the (2+1)-dimensional boiti-leon-pempinelli equation," *Applied Mathematics and Computation*, vol. 217, no. 4, pp. 1484–1490, 2010.

[10] W.-G. Feng, K.-M. Li, Y.-Z. Li, and C. Lin, "Explicit exact solutions for (2+1)-dimensional boiti-leon-pempinelli equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 5, pp. 2013–2017, 2009.

[11] M. Eslami, A. Neyrame, and M. Ebrahimi, "Explicit solutions of nonlinear (2+1)-dimensional dispersive long wave equation," *Journal of King Saud University - Science*, vol. 24, no. 1, pp. 69–71, 2012.

[12] M. Eslami and H. Rezazadeh, "The first integral method for Wu-Zhang system with conformable time-fractional derivative," *Calcolo*, vol. 53, no. 3, pp. 475–485, 2016.

[13] S. T. R. Rizvi and K. Ali, "Jacobian elliptic periodic traveling wave solutions in the negative-index materials," *Nonlinear Dynamics*, vol. 87, no. 3, pp. 1967–1972, 2017.

[14] H. M. Baskonus, T. A. Sulaiman, and H. Bulut, "On the novel wave behaviors to the coupled nonlinear Maccari's system with complex structure," *Optik*, vol. 131, pp. 1036–1043, 2017.

[15] H. M. Baskonus, T. A. Sulaiman, and H. Bulut, "New solitary wave solutions to the (2+1)-dimensional Calogero–Bogoyavlenskii–Schiff and the Kadomtsev–Petviashvili hierarchy equations," *Indian Journal of Physics*, vol. 91, no. 10, pp. 1237–1243, 2017.

[16] C. Cattani and A. Ciancio, "Hybrid two scales mathematical tools for active particles modelling complex systems with learning hiding dynamics," *Mathematical Models and Methods in Applied Sciences*, vol. 17, no. 2, pp. 171–187, 2007.

[17] M. H. Heydari, M. R. Hooshmandasl, G. B. Loghmani, and C. Cattani, "Wavelets Galerkin method for solving stochastic heat equation," *International Journal of Computer Mathematics*, vol. 93, no. 9, pp. 1579–1596, 2016.

[18] A. R. Seadawy, "Fractional solitary wave solutions of the nonlinear higher-order extended KdV equation in a stratified shear flow: Part I," *Computers and Mathematics with Applications*, vol. 70, no. 4, pp. 345–352, 2015.

[19] A. R. Seadawy, "Ion acoustic solitary wave solutions of two-dimensional nonlinear Kadomtsev-Petviashvili-Burgers equation in quantum plasma," *Mathematical Methods in the Applied Sciences*, vol. 40, no. 5, pp. 1598–1607, 2017.

[20] A. Yokus, H. M. Baskonus, T. A. Sulaiman, and H. Bulut, "Numerical simulation and solutions of the two-component second order KdV evolutionary system," *Numerical Methods for Partial Differential Equations*, vol. 34, no. 1, pp. 211–227, 2018.

[21] J. Liu, M. Eslami, H. Rezazadeh, and M. Mirzazadeh, "Rational solutions and lump solutions to a non-isospectral and generalized variable-coefficient Kadomtsev–Petviashvili equation," *Nonlinear Dynamics*, vol. 95, no. 2, pp. 1027–1033, 2019.

[22] N. Zhang and T.-C. Xia, "A new negative discrete hierarchy and its N-fold darboux transformation," *Communications in Theoretical Physics*, vol. 68, no. 6, pp. 687–692, 2017.

[23] H. M. Baskonus, T. A. Sulaiman, and H. Bulut, "Novel complex and hyperbolic forms to the strain wave equation in microstructured solids," *Optical and Quantum Electronics*, vol. 50, no. 14, 2018.

[24] B. G. Konopelchenko and V.G. Dubrovsky, "Some new integrable nonlinear evolution equations in (2+1) dimensions," *Physics Letters A*, vol. 102, no. 1-2, pp. 15–17, 1984.

[25] H. M. Baskonus and H. Bulut, "On the complex structures of Kundu-Eckhaus equation via improved Bernoulli sub-equation function method," *Waves in Random and Complex Media*, vol. 25, no. 4, pp. 720–728, 2015.

[26] F. Dusunceli, "Solutions for the drinfeld-sokolov equation using an IBSEFM method," *MSU Journal of Science*, vol. 6, no. 1, pp. 505–510, 2018.

[27] Z. Sheng, "Further Improved F-expansion method and new exact solutions of KD equation," *Chaos, Solitons and Fractals*, vol. 32, no. 4, pp. 1375–1383, 2007.

[28] A.-M. Wazwaz, "New kinks and solitons solutions to the (2+1) dimensional KD equation," *Mathematical and Computer Modelling*, vol. 45, no. 3-4, pp. 473–479, 2007.

[29] S. Kumar, A. Hama, and A. Biswas, "Solutions of KD equation by Traveling wave hypothesis and lie symmetry approach," *Applied Mathematics and Information Sciences*, vol. 8, no. 4, pp. 1533–1539, 2014.

[30] L. Song and H. Zhang, "New exact solutions for the Konopelchenko-Dubrovsky equation using an extended Riccati equation rational expansion method and symbolic computation," *Applied Mathematics and Computation*, vol. 187, no. 2, pp. 1373–1388, 2007.

[31] W.-G. Feng and C. Lin, "Explicit exact solutions for the (2+1) dimensional KD equation," *Applied Mathematics and Computation*, vol. 210, no. 2, pp. 298–302, 2009.

[32] A. Bekir, "Applications of the extended tanh method for coupled nonlinear evolution equations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 13, no. 9, pp. 1748–1757, 2008.

[33] T. Xia, Z. Lü, and H. Zhang, "Symbolic computation and new families of exact soliton-like solutions of Konopelchenko-Dubrovsky equations," *Chaos, Solitons and Fractals*, vol. 20, no. 3, pp. 561–566, 2004.


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